Dispersion in the Offset in fractal Dimension of Large Scale Structures

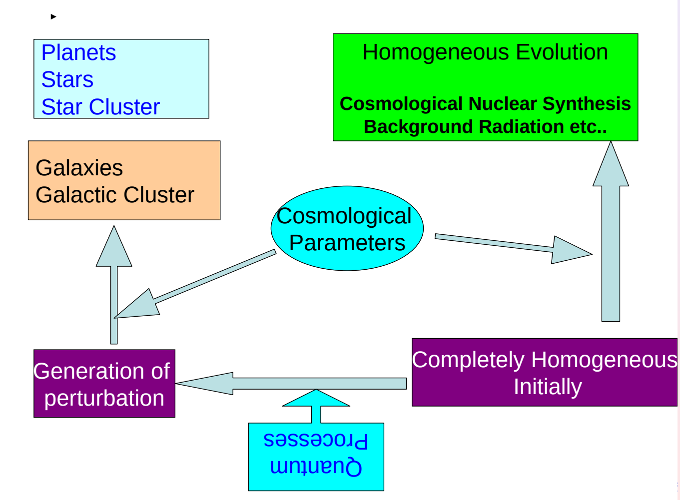
Jaswant K. Yadav[1]

Harish-Chandra Research Institute
Allahabad

ICCGF 09, IIT Kanpur, October 31, 2009

[1]In Collaboration with J.S. Bagla and N. Khandai

Pictorial Summary : Evolution of Universe



Outline

1. Large Scale Structures (LSS)
   - Properties
   - Distribution in the Sky

Outline

1. Large Scale Structures (LSS)
   - Properties
   - Distribution in the Sky
2. Statistical Measures of LSS
   - Issues Involved and Aims
   - Correlation Function
   - Fractal Dimension

Outline

Outline

LSS of Universe

- Structures of the scale of Mpc and higher. 1 Parsec $= 3 * 10^{18}$cm

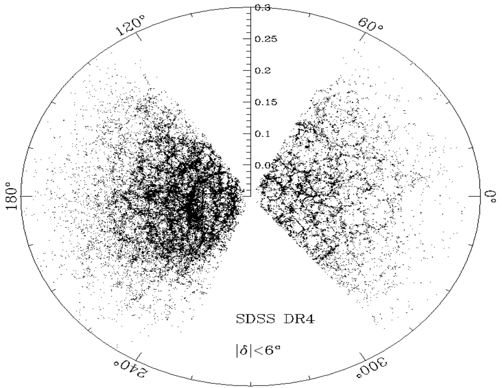
LSS of Universe

- Structures of the scale of Mpc and higher. 1 Parsec $= 3 * 10^{18}$cm
- Formed due to the presence of small inhomogeneities in the early Universe

LSS of Universe

- Structures of the scale of Mpc and higher. 1 Parsec $= 3 * 10^{18}$cm
- Formed due to the presence of small inhomogeneities in the early Universe
- Evolved primarily due to gravitational interaction among matter particles

LSS Distribution



Issues with statistical Analysis

- Definition of Statistical method and the analysis of the assumptions implicitly used

Issues with statistical Analysis

- Definition of Statistical method and the analysis of the assumptions implicitly used
- Construction of samples and consideration of cosmological corrections

Issues with statistical Analysis

- Definition of Statistical method and the analysis of the assumptions implicitly used
- Construction of samples and consideration of cosmological corrections
- Comparison of results in galaxy catalog with model predictions

Aims of Statistical Analysis

* Cosmographical Description of LSS.
* Physics of structure formation
  - Direct relation with nature of primordial fluctuation
  - Knowledge about DM distribution with the help of bias.
* The scale of crossover to homogeneity
* Form of galaxy correlation on small scales
* Complete Statistical Information about the distribution

Two Point Correlation function

In a homogeneous distribution

$$dP = \frac{\bar{n}}{N} dV,$$

Hence

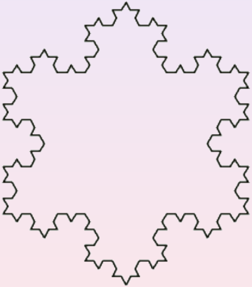
$$dP_{12} \propto dP_1 dP_2,$$

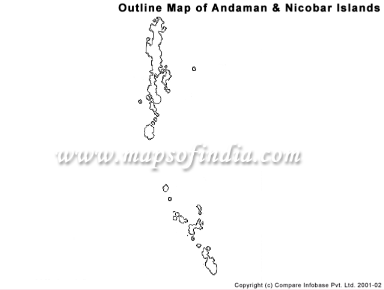
In a clustered distribution

$$dP_{12} = \left(\frac{\bar{n}}{N}\right)^2 \left[1 + \xi(\vec{r}_1, \vec{r}_2)\right] dV_1 dV_2$$

Fractals

Fractal is an object made of parts similar to the whole structure in some sense.





These are irregular objects defying the geometric measures.

Fractal Dimenson

- For strictly self similar objects, Dimension D is

$$N = S^{-D}$$

N = No of miniatures ; S = Scale factor Fractals are the objects which have fractional dimension.

- For strictly not self similar objects

$$N(r) \sim r^{-D}$$

N(r) : No. of hyper-cubes required to do covering

Correlation Dimenson and Generalisation

Correlation integral :

$$C_2(r) = \frac{1}{N^2} \sum_{i=1}^{N} n_i(r)$$

The Dimension D_2, then is :

$$C_2(r) \sim r^{D_2}$$

Generalising the relation we get :

$$C_q(r) = \frac{1}{NM} \sum_{i=1}^{M} n_i^{q-1}(r)$$

The generalised Dimension D_q, then is :

$$C_q(r) \sim r^{D_q(q-1)}$$

Minkowski-Bouligand Dimension

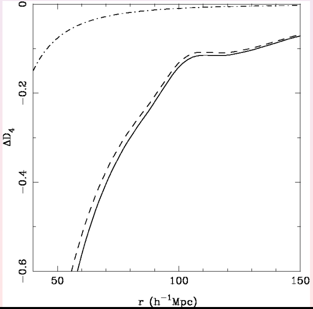
$$D_q = \frac{1}{q-1}\frac{\partial \log C_q}{\partial \log r}$$

1. Different in different range of scales
2. $q = 1, 2$ are Box counting dimension the correlation dimension
3. Complete statistical information by way of all the higher correlation function
4. $D_q = D$ for a monofractal
5. Information about regions with various amount of clustering

Minkowski-Bouligand Dimension for a Weakly-Clustered Distribution

$$D_q(r) = D - \frac{D(q-2)}{\bar{N}} - \frac{Dq}{2}\left(\bar{\xi}(r) - \xi(r)\right)$$

$$\Delta D_q = -(\Delta D_q)_{\bar{N}} - (\Delta D_q)_{clus}$$



With
$b = 2, \bar{N} = 5 \times 10^{-5}$

Bagla et. al. (2008),MNRAS,390, 829

Dispersion in ΔD_q

$$Var\{Dq\} \simeq Var\{(\Delta D_q)_{clus}\}$$

This implies that

$$\begin{aligned}
Var\{\Delta D_q\} &\simeq Var\{\frac{Dq}{2}(\bar{\xi}(r) - \xi(r))\} \\
&\simeq \left(\frac{Dq}{2}\right)^2 \left(Var\{\bar{\xi}(r)\} + Var\{\xi(r)\} + 2Cov\{\bar{\xi}(r)\xi(r)\}\right) \\
&\simeq \left(\frac{Dq}{2}\right)^2 \left(Var\{\bar{\xi}(r)\} + Var\{\xi(r)\}\right. \\
&\qquad \left. +2\sqrt{Var\{\bar{\xi}(r)\}Var\{\xi(r)\}}\right) \\
&\simeq \left(\frac{Dq}{2}\right)^2 \left(\sqrt{Var\{\bar{\xi}(r)\}} + \sqrt{Var\{\xi(r)\}}\right)^2
\end{aligned}$$

Variance of Power Spectrum and Correlation Function

FKP (1994) propsed

$$\sigma_P(k) = \sqrt{\frac{2}{V}}\left(P(k) + \frac{1}{\bar{n}}\right),$$

With This

$$Cov_\xi(r, r') = \int \frac{\mathrm{d}k\, k^2}{2\pi^2} j_0(kr) j_0(kr') \sigma_P^2(k)$$

We can relate this to

$$Cov_{\bar{\xi}}(i, j) = \frac{1}{V_i\, V_j} \int \mathrm{d}^3 r \int \mathrm{d}^3 r' \, Cov_\xi(r, r')$$

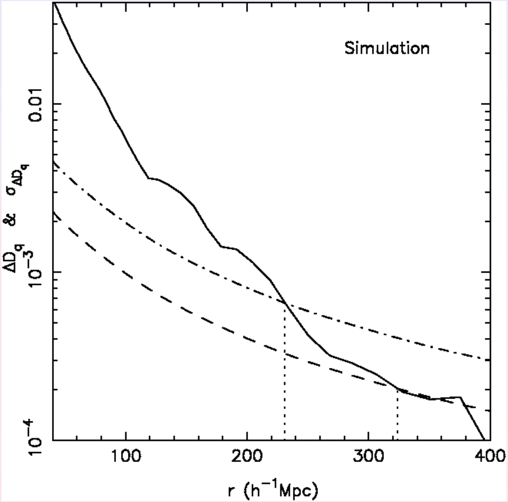
$$= \int \frac{\mathrm{d}k\, k^2}{2\pi^2} \bar{j}_0(k, i) \bar{j}_0(k, j) \sigma_P^2(k)$$

Model Predictions

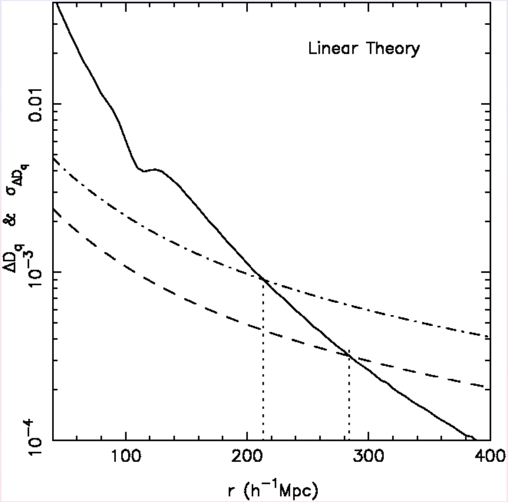
The *scale of homogeneity* can be defined as the scale above which $\sigma_{\Delta D_q}$ exceeds ΔD_q

- As long as non-linear correction are not important, the scale of homogeneity does not change with epoch.
- In real space, the scale of homogeneity is independent of the tracer used as the deviation ΔD_q as well as the dispersion in this quantity scale in the same manner with bias.
- Redshift space distortions introduce some bias dependance in the scale of homogeneity.
- As long as our assumption of $q\bar{\xi} \ll 1$ is valid, the scale of homogeneity is the same for all q.

$N-$Body Simulations



Linear Theory



Simulation + Missing Modes