

# Optimal Frame Rate Allocation for Unicast and Multicast Wireless Video Communication

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**Abstract**—In this paper we consider a paradigm for sum perceptual quality maximization in the context of unicast and multicast video transmission to heterogeneous multimedia wireless clients. The multimedia server is constrained by the processing overheads required for tasks such as video encoding, bit-stream extraction, packetisation etc. We demonstrate that the above problem of perceptual quality maximization is well represented by a constrained optimization framework. Further, this model can be readily extended to include QoS considerations in multicast transmission. Based on the system model, we present a closed form solution for frame rate allocation and a comprehensive algorithm for sum quality maximization. We compare the results obtained using the above mentioned frame rate allocation with the results obtained using a content agnostic equal frame rate allocation scheme and demonstrate the superiority of the proposed algorithm in both unicast and multicast scenarios.

## I. INTRODUCTION

Next generation rich mobile applications target vast services based on multimedia content with utilization in diverse fields of interest. There is a significant demand for applications such as High Definition (HD) video streaming, online 3-D gaming, multiparty video conferencing, surveillance, etc. 4G wireless communication technologies such as LTE, WiMAX and UMTS are characterized by applications involving high quality and reliable delivery of multimedia content, real time video streaming, virtual reality experiences and high definition mobile television. Thus, wireless video communications is inseparable from the context of 4 G wireless technologies [1], [2]. The emerging trend towards progressive miniaturization has motivated system designers to work towards offloading the computational complexity of end users and embedding more intelligence/ processing power in centralized servers/ cluster heads owned by the service provider [3]. This enables thin wireless clients to support powerful multimedia applications. Consider, for instance a cellular telephony network as shown in Fig.1.

The media server stores, processes and streams the multimedia content to the clients. This server is housed together with the Base Transceiver Station (BTS) which transmits to the clients on wireless links[4]. The media server is assumed to be connected on extremely high speed dedicated fiber optic links with its backhaul network which provides the server with the required media content for diverse areas of interest. Thus,

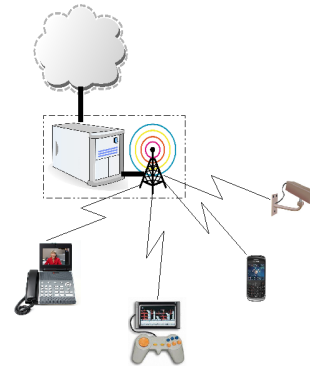


Fig. 1. Example Wireless Communication Scenario

the server is constrained to satisfy the variegated demands for multimedia services originating from a plethora of devices belonging either to the unicast or multicast category. Further, this central access server is fundamentally constrained by its processing capacity for tasks such as multimedia coding and compression, packetisation, HARQ based retransmission etc. Hence, it is essential to optimally allocate the available computing resources so as to maximize the end-user multimedia service experience while also adhering to potentially graded QoS criteria for different services.

In this context, we consider a novel paradigm for resource allocation based on perceptual quality maximization. Recently there has been a significant amount of research relating to parametric characterization of perceptual quality [5]. Perceptual quality based utility function has the advantage that it directly relates to the end user experience for video quality. Several other measures employed in conventional works such as MSE, and other distortion functions are not directly related to perceptual quality. Hence maximization of perceptual quality directly relates to the end user experience. The perceptual quality metric for video sequences has been modeled as a function of its frame rate. We employ this function to maximize sum of perceptual qualities of the video sequences requested by heterogeneous clients in the vicinity of a media server subject to overall processing constraints of the server using the robust framework of optimization [6]. The optimal solution thus leads

us to a set of frame rates at which the scalable video streams corresponding to the required video sequences can be coded and transmitted to the clients.

The proposed optimal resource allocation scheme has a superior performance compared to equal frame rate allocation. We demonstrate that this framework can naturally be extended to a Multicast scenario where the media server is required to serve video sequences to various multicast groups [7]. Here again the optimal frame rate allocation scheme comes out to be better than equal frame rate allocation. We also verify the results of the optimal frame rate allocation scheme with a Matlab based convex solver CVX [8]. Further, as is shown in the paper, this framework can also be extended to include graded priority of the different streams pertaining to Quality of Service constraints and address issues related to admission control.

The rest of this paper is organized as follows. In Section II we formulate the optimal frame rate allocation problem. Section III contains a closed form solution of the problem. In Section IV, we compare the performance of the optimal frame rate allocation scheme with that of equal frame rate allocation for various unicast and multicast scenarios. Finally we conclude in Section V.

## II. WIRELESS VIDEO SYSTEM MODEL

In this section we begin with an analytical characterization of the unicast multicast framework for video transmission to multimedia clients. Consider a video sequence of maximum frame rate  $f_{max}$ . The normalized perceptual quality of a video sequence has been accurately modeled in [5] by the following function of the variable frame rate  $f$ ,

$$SQ(f) \triangleq \left( \frac{1 - e^{-c \frac{f}{f_{max}}}}{1 - e^{-c}} \right), \quad (1)$$

where  $f_{max}$  is the highest frame rate at which the video sequence is encoded using scalable video coding. The model parameter  $c$  is dependent on the video sequence. It determines the degradation of perceptual quality as the frame rate decreases in (1). This model parameter  $c$  has been determined for known test sequences by conducting subjective tests and using a least squares fitting method. Further, a relationship of this exponential parameter with various features of the video sequence has been explored. Thus, the authors in [5] have determined that the value of the model parameter  $c$  is high for slow moving video sequences and it decreases linearly as motion and contrast contents in a video sequence increase with a high Pearson correlation coefficient of  $-0.93$ . Therefore, given a video sequence, the value of  $c$  can be objectively determined. Hence, one can compute the model parameter  $c$  for the diverse range of  $N$  video sequences present with the media server, each sequence characterized by the parameter  $c_i$ ,  $1 \leq i \leq N$ . We set our extreme references to standard test sequences obtained from [9] Akiyo (Fig.2) with value of  $c = 9.82$  and Football (Fig.3) with value of  $c = 5.79$ .

Processing of these video sequences while serving them to the clients results in increasing the load on the server. It was



Fig. 2. Akiyo test sequence, CIF resolution (352x288) at 30 frames/sec



Fig. 3. Football test sequence, CIF resolution (352x288) at 30 frames/sec

shown in [3] that processing load incurred by a video sequence can be modelled as piecewise affine function of the frame rate with positive slopes. It was also shown that processing load is greater for video sequence with high motion content when compared with slow moving sequences at equal frame rates. We set the minimum frame rate ( $f_{min}$ ) to be served as 17  $fps$  due to perceptual continuity of Human Visual System [10] and maximum frame rate ( $f_{max}$ ) as 30  $fps$ . We thus assign processing load for each video sequence  $c_i$ ,  $1 \leq i \leq N$  at frame rates 15, 20 and 25  $fps$  as  $p_{i1}$  and  $p_{i2}$  and  $p_{i3}$ ,  $1 \leq i \leq N$  respectively in terms of percentage of processing capacity of media server. Employing processing loads at 15, 20 and 25  $fps$ , these can be converted to piecewise linear functions of frame rate as,

$$\begin{aligned} (a_{i1}f + b_{i1}), & \quad f_{min} \leq f \leq 20 \\ (a_{i2}f + b_{i2}), & \quad 20 \leq f \leq f_{max} \end{aligned}$$

The above mentioned piece-wise linear function can be reformulated as,

$$P(f) = \max\{a_{i1}f + b_{i1}, a_{i2}f + b_{i2}\}, \quad 1 \leq i \leq N \quad (2)$$

Thus, (2) corresponds to the piecewise affine function  $P(f)$  which determines the processing load incurred by the  $i^{th}$

video sequence in terms of percentage of the total processing capacity of the media server.

#### A. Unicast System Model

We now formulate a Unicast scenario where video sequences are streamed to various users over dedicated wireless channels. The total normalized perceptual quality of all the video sequences being serviced at any instant is the objective function which we aim to maximize, subject to being constrained by the overall processing capacity  $F$  of the media server and keeping the frame rate for all sequences bounded between  $f_{min}$  and  $f_{max}$ . This problem can be expressed as,

$$\begin{aligned} \min. & - \sum_{i=1}^N \frac{\left(1 - e^{-c_i \frac{f_i}{f_{max}}}\right)}{(1 - e^{-c_i})} \\ \text{subject to} & \sum_{i=1}^N \max\{a_{i1}f + b_{i1}, a_{i2}f + b_{i2}\} \leq F \\ & f_i \leq f_{min}, \quad 1 \leq i \leq N \\ & f_i \geq f_{max}, \quad 1 \leq i \leq N \end{aligned}$$

Utilizing the convexity properties of the piecewise linear constraint, we convert it into a finite set of linear inequalities. The above optimization problem can thus be equivalently expressed as,

$$\begin{aligned} \min. & - \sum_{i=1}^N \frac{\left(1 - e^{-c_i \frac{f_i}{f_{max}}}\right)}{(1 - e^{-c_i})} \\ \text{subject to} & \sum_{i=1}^N t_i \leq F \\ & (a_{i1}f + b_{i1}) \leq t_i, \quad 1 \leq i \leq N, \quad f_{min} \leq f \leq 20 \\ & (a_{i2}f + b_{i2}) \leq t_i, \quad 1 \leq i \leq N, \quad 20 < f \leq f_{max} \\ & f_i \leq f_{min}, \quad 1 \leq i \leq N \\ & f_i \geq f_{max}, \quad 1 \leq i \leq N \end{aligned}$$

The above problem can be readily solved by an interior point method. However, to make the problem tractable and to illustrate a closed form solution in this work, we assume the processing load to be a single affine function of the frame rate restricted within the bounds  $[f_{min}, f_{max}]$ . It can be noted that both the simplified problem and the problem stated above are optimization problems with convex objective function and linear inequality constraints, with identical computational complexities. The convex optimization problem for the unicast scenario can thus be reformulated as,

$$\begin{aligned} \min. & - \sum_{i=1}^N \frac{\left(1 - e^{-c_i \frac{f_i}{f_{max}}}\right)}{(1 - e^{-c_i})} \\ \text{subject to} & \sum_{i=1}^N (a_i f + b_i) \leq F \\ & f_i \leq f_{min}, \quad 1 \leq i \leq N \\ & f_i \geq f_{max}, \quad 1 \leq i \leq N \end{aligned} \quad (3)$$

#### B. Multicast System Model

The framework described in previous sub-section can be naturally extended to a Multicast scenario where each video sequence can be requested by  $n_i$  users,  $1 \leq i \leq N$ . We also incorporate an additional parameter, weight ( $w_i$ ,  $1 \leq i \leq N$ ) for each Multicast Group. The weight assigned to a multicast group signifies the QoS priority assured by the service provider to that multicast group. It is apparent that only the objective function of the optimization problem changes in this scenario where as the constraint functions continue to remain unchanged from the unicast scenario. The objective function for the multicast scenario can be expressed as

$$\min. - \sum_{i=1}^N n_i w_i \frac{\left(1 - e^{-c_i \frac{f_i}{f_{max}}}\right)}{(1 - e^{-c_i})} \quad (4)$$

### III. OPTIMAL FRAME RATE ALLOCATION SCHEME

The function  $e^{-x}$  is a convex function and  $\forall x \geq 0$  it is bounded between  $[1, 0)$ . Therefore, the function  $-(1 - e^{-x})$ ,  $\forall x \geq 0$  is a convex function bounded between  $[0, -1)$ . Using the fact that sum of convex functions preserves convexity of the objective, it can be shown that the objective functions in (3) and (4) are convex functions. All  $(2N + 1)$  inequality constraints are affine (convex) functions of frame rate. Therefore the optimization problems stated in previous section are convex optimization problems and can be solved by fast convex solvers. As demonstrated above, even by considering the processing capacity constraint as piecewise linear, the problems will continue to remain convex with additional linear constraints. Therefore, a globally unique solution to both the problems can be determined using the robust convex optimization framework. We now derive a closed form solution for the multicast system model using a procedure involving the Lagrangian below.

#### A. Admission Control

Naturally, in a broadband wireless network, it is essential to implement a policy of admission control so as to satisfy all the users at a minimum QoS level. Hence, we propose a criterion that the media server prior to accepting a service request from a new multicast group will confirm that,

$$\sum_{i=1}^K (a_i f_{min} + b_i) + (a_j f_{min} + b_j) \leq F \quad (5)$$

where there are  $K$  multicast groups being currently served and the  $j^{th}$  multicast group is requesting for a different video sequence. If (5) is not met, then the new multicast group is denied admission. It can be readily seen that a feasible point exists if the admission control condition in (5) is satisfied.

#### B. Optimal Solution using Lagrangian Multipliers

In this section we derive expressions for the optimal frame rate allocation based on the above convex optimization problem. The Lagrangian for the optimization problem mentioned

in (4) can be formulated as,

$$L(\mathbf{f}, \lambda, \mu, \gamma) = - \sum_{i=1}^N n_i w_i \frac{\left(1 - e^{-c_i \frac{f_i}{f_{max}}}\right)}{\left(1 - e^{-c_i}\right)} + \lambda \left( \sum_{i=1}^N (a_i f_i + b_i) - F \right) + \sum_{i=1}^N \mu_i (f_i - f_{max}) + \sum_{i=1}^N \gamma_i (f_{min} - f_i)$$

The KKT conditions, which can be readily derived as,

$$\begin{aligned} -n_i w_i \frac{c_i}{f_{max}} \left( \frac{e^{-c_i \frac{f_i}{f_{max}}}}{1 - e^{-c_i}} \right) + a_i \lambda + \mu_i - \gamma_i &= 0 \\ \lambda \geq 0, \sum_{i=1}^N (a_i f_i + b_i) \leq F, \lambda \left( \sum_{i=1}^N (a_i f_i + b_i) - F \right) &= 0 \\ \mu_i \geq 0, f_i \leq f_{max}, \mu_i (f_i - f_{max}) &= 0 \\ \gamma_i \geq 0, f_i \geq f_{min}, \gamma_i (f_{min} - f_i) &= 0 \end{aligned}$$

can be employed to compute the optimal solution. If  $\mu_i, \gamma_i = 0$ , the optimal solution can be derived as below (else, one can employ the algorithm that follows next).

$$\begin{aligned} a_i \lambda &= n_i w_i \frac{c_i}{f_{max}} \left( \frac{e^{-c_i \frac{f_i}{f_{max}}}}{1 - e^{-c_i}} \right) \\ f_i^* &= - \frac{f_{max}}{c_i} \log \left( \frac{a_i \lambda f_{max} (1 - e^{-c_i})}{n_i w_i c_i} \right) \end{aligned} \quad (6)$$

Using the sum constraint  $\sum_{i=1}^N a_i f_i + b_i = F$ , the parameter  $\lambda$  can be derived as,

$$\begin{aligned} F &= - \sum_{i=1}^N \left( a_i \frac{f_{max}}{c_i} \log \left( \frac{\lambda a_i f_{max} (1 - e^{-c_i})}{n_i w_i c_i} \right) + b_i \right) \\ F - \sum_{i=1}^N b_i &= - \sum_{i=1}^N a_i \frac{f_{max}}{c_i} \log \lambda \\ &\quad - \sum_{i=1}^N a_i \frac{f_{max}}{c_i} \log \left( \frac{a_i f_{max} (1 - e^{-c_i})}{n_i w_i c_i} \right) \end{aligned}$$

Hence, the final expression for the Lagrangian dual variable  $\lambda^*$  can be derived as,

$$\exp \left( \frac{F - \sum_{i=1}^N b_i + \sum_{i=1}^N a_i \frac{f_{max}}{c_i} \log \left( \frac{a_i f_{max} (1 - e^{-c_i})}{n_i w_i c_i} \right)}{- \sum_{i=1}^N a_i \frac{f_{max}}{c_i}} \right) \quad (7)$$

Substitution of the above value for  $\lambda^*$  in (6) gives the desired optimal frame rate for the  $i^{th}$  multicast group.

### C. Algorithm for Computing Optimal Frame Rate

In this section, we present an algorithm to determine the optimal frame rate as a solution to the above convex optimization problem. We initialize a multicast scenario with

$(N - 1)$  multicast groups, presently being serviced by the media server and an  $N^{th}$  multicast group places a request for a video sequence. The media server determines optimal frame rate using the procedure described in Algorithm 1

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### Algorithm 1 Computation of optimal frame rate

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**begin**

**if** Admission control condition (5) is satisfied

    Compute frame rates  $f_i$ ,  $1 \leq i \leq N$  for each multicast group using (6);

**for** ( $i = 1 : 1 : N$ )

    { **if**  $f_i > f_{max}$

        fix the frame rate for  $i^{th}$  multicast group as  $f_{max}$ ;

        compute  $\tilde{F} = F - (a_i f_{max} + b_i)$ ;

        compute  $\lambda$  using  $\tilde{F}$  in (7) and compute frame rates for  $(N - i^{th})$  multicast groups using (6);

        repeat **for** loop for  $(N - i^{th})$  multicast groups;

**else**

**if**  $f_i < f_{min}$

            fix the frame rate for  $i^{th}$  multicast group as  $f_{min}$ ;

            compute  $\tilde{F} = F - (a_i f_{min} + b_i)$ ;

            compute  $\lambda$  using  $\tilde{F}$  in (7) and compute frame rates for  $(N - i^{th})$  multicast groups using (6);

            repeat **for** loop for  $(N - i^{th})$  multicast groups;

**end if**

**end if** }

**end if**

**end**

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Successful termination of the **for** loop in Algorithm 1 will yield the optimal frame rates for all multicast groups.

### D. Optimal Frame Rate Allocation - Unicast Scenario

Setting the values of  $n_i = w_i = 1$  in (4) and proceeding similarly, a closed form solution for the optimization problem for the unicast scenario described in (3) can also be obtained. It can be shown that the value of Lagrangian dual variable  $\lambda^*$  can be computed using the following expression,

$$\exp \left( \frac{F - \sum_{i=1}^N b_i + \sum_{i=1}^N a_i \frac{f_{max}}{c_i} \log \left( \frac{a_i f_{max} (1 - e^{-c_i})}{c_i} \right)}{- \sum_{i=1}^N a_i \frac{f_{max}}{c_i}} \right)$$

Substitution of the above value for  $\lambda^*$  in the equation given below yields the desired optimal frame rate for the  $i^{th}$  user.

$$f_i^* = - \frac{f_{max}}{c_i} \log \left( \frac{a_i \lambda^* f_{max} (1 - e^{-c_i})}{c_i} \right)$$

All the rest of the steps including the algorithm are similar to that of the multicast scenario.

## IV. SIMULATION AND COMPARISON OF RESULTS

We compare the performance of our optimal allocation with that of equal frame rate allocation in which all users/multicast groups requesting for video sequences are allotted equal frame rate which is calculated based on the sequence with the worst case processing load on the media server. Further, to verify the

optimality of our algorithm, we compare the allocation with that of CVX solver [8] in our simulations. We test the unicast scenario in simulation 1 where the media server contains sequences of diverse video content. The results are plotted in Fig.4 and Fig.5.

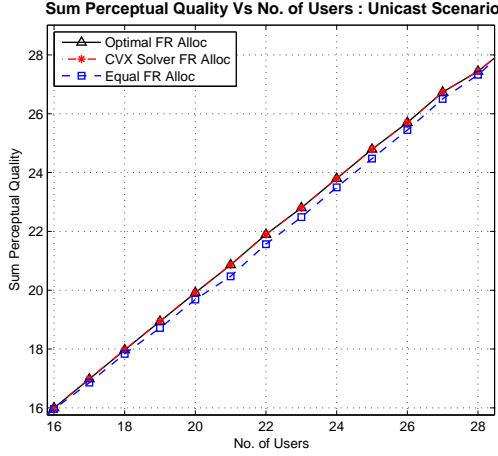


Fig. 4. Unicast Scenario - Diverse Sequence Collection

Analyzing the results of simulation 1, we observe that the total

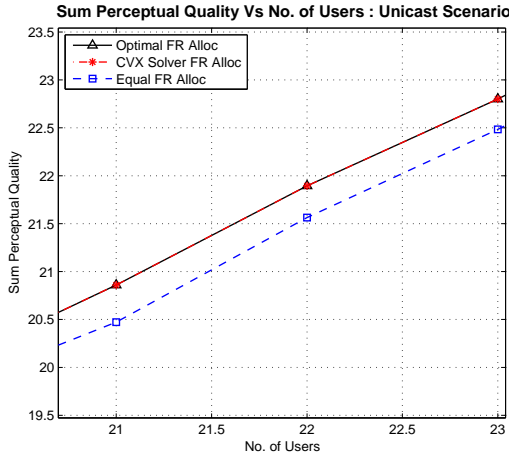


Fig. 5. Unicast Scenario - Diverse Sequence Collection (Close Up)

normalized perceptual quality calculated through the convex optimization solver confirms our optimal frame rate allocation scheme over the complete range.

The total normalized perceptual quality calculated through equal frame rate allocation scheme is at par with the same calculated using optimal frame rate allocation up till 14 users beyond which the load on the server increases and optimal frame rate allocation scheme proves to be better than equal frame rate allocation scheme. It is also observed that the total normalized perceptual quality calculated through optimal frame rate allocation scheme tends to become equal to the same calculated using equal frame rate allocation beyond 28 users. This is due to high load on the server, almost all

the frame rates obtained using optimal frame rate allocation scheme approach  $f_{min}$ . The admission control check condition does not permit more than a specific number of users to load the server.

Fig.5 brings out clearly that optimal frame rate allocation scheme is better than equal frame rate allocation. The separation between two total normalized perceptual qualities is maximum when the users utilizing the system are in the range of 21 to 24. This can be deduced as the optimal load on the server.

We simulate the unicast scenario in simulation 2 once again where the media server contains sequences of specific video content (only very high motion e.g. sports footages and only very low motion e.g. News sequences). The results obtained using both allocation schemes and by the convex optimization solver are plotted in Fig.6.

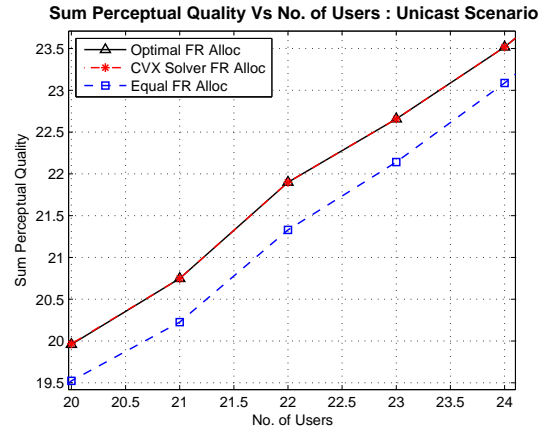


Fig. 6. Unicast Scenario - Specific Sequence Collection(Close Up)

It can be clearly observed that the gap between total normalized perceptual quality obtained via optimal frame rate allocation scheme and the same obtained via equal frame rate allocation scheme is better in simulation 2 than in simulation 1. Therefore we can claim that optimal frame rate allocation scheme maximizes the total normalized perceptual quality even better when the media server is being utilized for tasks such as a news server.

We test the multicast scenario in simulation 3 where the media server contains sequences of diverse video content. We utilize the input gained in simulation 1 that the server is optimally loaded when 21-24 sequences are being served. We keep the number of multicast groups constant at 22 where as increase the number of users subscribing these groups in 10 iterations. The results obtained are as shown in Fig.7.

It is observed from Fig.7 that the solution obtained through the Convex Solver completely verifies the same produced by our optimal frame rate allocation scheme. The gap between total normalized perceptual quality obtained via optimal frame rate allocation scheme and the same obtained via equal frame rate allocation scheme widens as number of users increase. This proves that optimal frame rate allocation scheme fairs better for multicast scenarios.

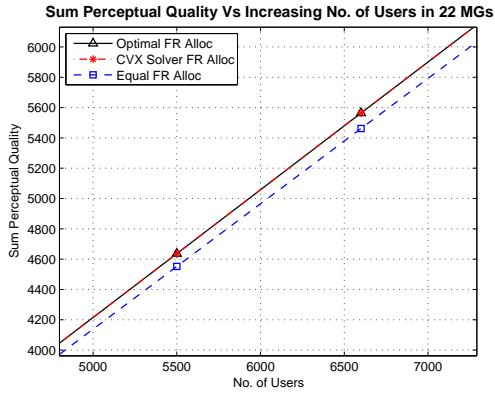


Fig. 7. Multicast Scenarios - 22 multicast groups with increasing users

We conduct another test in the multicast scenario in simulation 4 where we fix the number of users in each multicast group as 1000 and similar to simulation 1 plot the total normalized perceptual quality as the media server is loaded progressively with a multicast group requesting a unique video sequence joins iteratively. The results obtained using both allocation schemes and by the convex solver are plotted in Fig.8 and Fig.9.

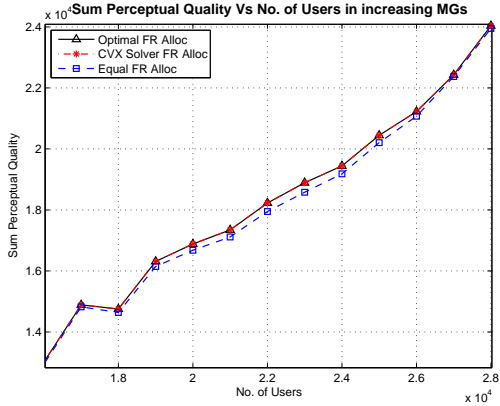


Fig. 8. Multicast Scenarios - increasing multicast groups

The results obtained are similar to that obtained through simulation 1. The gap between the total normalized perceptual quality is maximum when the active multicast groups are 21-24. The zigzag nature of the curve is due to the random weight function which makes the sum perceptual quality shoot up when a multicast group having a significant priority joins, whereas the rise is not very substantial when a multicast group with less priority is added to be serviced by the media server.

## V. CONCLUSION

In this paper we proposed a frame rate allocation problem for video streaming services to multiple wireless clients. We motivated the relevance of this problem in the context of 4G wireless communications and beyond. Employing the sum

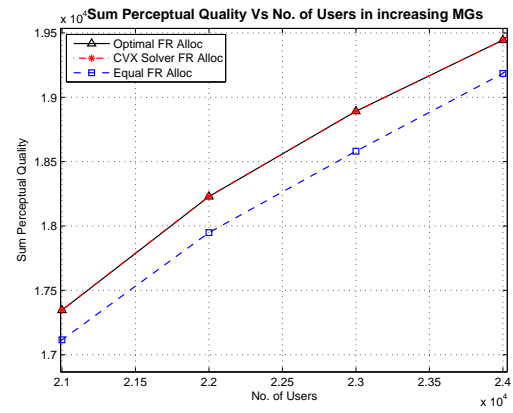


Fig. 9. Multicast Scenarios - increasing multicast groups (Close Up)

perceptual quality metric as a function of the frame rate vector, we demonstrated that this scenario can be well captured by an optimization framework with a convex objective function. Further, this framework can be readily extended to multicast scenarios with graded priority. A closed form solution was obtained for the above problem. We conducted simulations in unicast and multicast scenarios and compared the results of the optimal frame rate allocation scheme with those of equal frame rate allocation. We confirmed our theoretical solution with the solution obtained using a convex solver. We deduced that over all the simulations being conducted the optimal frame rate allocation scheme comes out to be superior than equal frame rate allocation. The optimal frame rate allocation scheme performs even better in a multicast scenario as compared to a unicast scenario.

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