Hierarchical DWT Based Optimal Diversity Power Allocation for Video Transmission in 4G OFDMA Wireless Systems

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***Abstract*—In this paper we propose novel algorithms for optimal subcarrier power allocation towards video transmission in Orthogonal Frequency Division Multiple Access (OFDMA) based 4G wireless systems. A unique feature of the proposed schemes is that they employ diversity dependent subcarrier power distribution relying on a discrete wavelet transform (DWT) based hierarchical video decomposition. Another key aspect of the schemes is that they exclusively employ partial channel state information (CSI) feedback based on the order statistics of the allotted subcarriers in the OFDMA system. This significantly reduces the communication overhead required on the reverse link, thus reducing complexity and enhancing throughput. It is demonstrated that the paradigm of ordered subcarrier diversity based video distortion minimization can be formulated as a constrained convex cost minimization problem. Further, we illustrate that this cost minimization reduces to a polynomial root computation under a suitable approximation. Closed form expressions are derived for the diversity based optimal power allocation scheme. Simulation results employing several video sequences for an OFDMA system demonstrate superior performance of the proposed optimal subcarrier power allocation schemes over equal power allocation.**

I. Introduction

With the advent of fourth generation (4G) wireless standards such as WiMAX and LTE, broadband video transmission with robust quality of service(QoS) has become a reality. Reliable video communication is essential for key applications such as surveillance, video conferencing, mobile gaming etc. Orthogonal Frequency Division Multiple Access (OFDMA) is the most attractive physical layer technology for 4G broadband wireless networks due to its robustness to inter symbol interference arising from multipath interference combined with a low complexity IFFT/FFT based implementation. Hence, it is suitable for transmitting high data rates over wireless links, one of the key requirements for video transmission. However, video transmission in OFDMA scenarios is challenging, particularly due to the fading nature of wireless channels resulting in severe degradation of video quality.

To achieve high quality video transmission in an OFDMA system, it is critical to optimally distribute the limited power resources amongst the subcarriers. One such power allocation based on the water filling algorithm was examined in [1]. In [2] the authors propose an iterative power allocation method to minimize power usage in OFDM for multiple access channels. These are optimal solutions for transmitting generic data, and are not specifically targeted at minimizing the overall video distortion. Hence, one is motivated to develop optimal schemes, especially suited for video transmission.

Specific properties of the video stream can be exploited to minimize distortion in video transmission. Cross-layer resource allocation techniques, such as in [3], have been proposed for video communications. The scheme in [4] uses joint power allocation and antenna selection for scalable video delivery over MIMO OFDMA systems. In our work, we consider a novel ordered subcarrier diversity-order dependent power allocation relying on a DWT based hierarchical decomposition of the video stream. We motivate this paradigm for video transmission in wireless systems, since it naturally merges the hierarchical layering of video data with that of the graded reliability associated with the diversity order of the wireless channel.

We formulate the above problem as the minimization of a constrained convex video distortion objective function and demonstrate that the optimal power allocation problem reduces to one of polynomial root computation. Further, the proposed scheme employs only partial CSI feedback, which significantly reduces the overhead on the reverse link, thus making it attractive for practical implementation. Simulation results for OFDMA based wireless transmission employing several video sequences demonstrate that the proposed schemes outperform equal power allocation in terms of both PSNR and visual quality.

We begin with a formulation of our diversity based subcarrier allocation scheme for OFDMA wireless systems in the next section and present a framework to characterize the probability of bit-error for the proposed transmission scheme. Subsequently, in section III we propose the diversity based hierarchical allocation (DHA) scheme for video transmission. This scheme employs DWT filtering for spatio-temporal hierarchical decomposition of the video sequence. This framework is utilized to formulate an optimization criterion based on a convex objective function for optimal power allocation (OPA) amongst the OFDMA subcarriers in section IV. A closed form power allocation (CFPA), with performance close to OPA and significantly lower computational complexity is also proposed. Simulation results are given in section V to illustrate the performance gains of the proposed DHA, OPA and CFPA schemes over equal power allocation (EPA).

II. ORDER DIVERSITY IN OFDMA

In this section, we propose a diversity based subcarrier characterization scheme for video transmission in OFDMA systems. Consider an OFDMA system of bandwidth W, for data transmission over a multipath Rayleigh fading channel with L independent fading multipath components. The time-domain baseband system model for the above system can be described for sampling instants $0 \leq m \leq N + L_{cp} - 1$ as,

$$y(m) = \sum_{l=0}^{L-1} h(l)x(m-l) + v(m),$$

where N is the number of subcarriers in the OFDMA system and L_{cp} is the cyclic prefix (CP) length. After CP removal and FFT at the receiver, the resulting system can be expressed as,

$$Y_k = H_k X_k + V_k, \qquad 0 \leq k \leq N-1 \tag{1}$$

where X_k and Y_k represent the data transmitted and received respectively through subcarrier k. It is known [5] that the subcarrier gains $[H_0, H_1, \ldots, H_{N-1}]^T$ for the above OFDMA system are obtained through an N-point DFT of the L-tap multipath wireless channel $\mathbf{h} \triangleq [h(0), h(1), \ldots, h(L-1)]^T$, where each path gain $h(i)$ is an independent Rayleigh fading complex channel coefficient. We denote by $[V_0, V_1, \ldots, V_{N-1}]^T$, the N-point DFT of the noise vector $\mathbf{v} = [v(0), v(1), \ldots, v(N-1)]^T$, where the noise $\mathbf{v} \sim \mathcal{N}(0, \sigma_v^2 \mathbf{I}_N)$ is complex additive white Gaussian with covariance matrix $E\left(\mathbf{v}\mathbf{v}^H\right) = \sigma_v^2 \mathbf{I}_N$.

In the context of the above model, we consider an OFDMA system with N subcarriers and K video users such that N_u subcarriers are allocated to each user. Note that $KN_u < N$, implying that there are other users in the system with applications not based on video transmission (such as voice, data etc.) occupying the rest of the subcarriers. To minimize the video distortion, it is critical to appropriately distribute layers of the video stream amongst the N_u subcarriers allotted to a particular user and further to optimally allocate power to each of the subcarriers.

Consider the N_u independent Rayleigh fading subcarriers allotted to a particular user as per the property described in section VII. The channel state information(CSI) related to the instantaneous fading state of the wireless channel is unknown to the transmitter. Explicit feedback of the channel coefficients $H(i)$ requires a large number of feedback bits. We therefore motivate an order feedback of indices $[i], i \in \{1, 2, ..., N_u\}$ such that $|H_{[i]}| > |H_{[j]}|$ for $i > j$. For this purpose, the N_u subcarrier gains for each user are ordered at the receiver as follows,

$$|H_{[N_u]}| \geq |H_{[N_u-1]}| \geq \cdots \geq |H_{[2]}| \geq |H_{[1]}|.$$

Subsequently, the receiver only feeds back this ordering information of the user subcarrier gains to the transmitter. Thus, since an elaborate feedback of the exact subcarrier gains is avoided, this scheme results in a significantly lower overhead on the reverse link. For instance, in the case of $N_u = 2$, only 1 bit is needed to be fed back by the receiver. Hence, our order statistics based partial feedback scheme results in a significant bit saving compared to a bit-expensive explicit feedback containing exact values of the channel coefficients. Below we present the closed form expression for the probability of bit-error for the above ordered subcarriers based transmission scheme.

Theorem 2.1: The bit-error rate (BER) corresponding to the $[n]^{th}$ ordered subcarrier has a diversity order n. Further, the probability of bit-error ϕ_e for binary phase-shift keyed (BPSK) data transmission through this subcarrier can be expressed as,

$$\phi_e\left(P_n\right) = \sum_{i=n}^{\infty} \alpha_{ni} \left(\frac{\sigma_v^2}{P_n}\right)^i. \tag{2}$$

where the coefficient α_{ni} in the above expression is given as,

$$\alpha_{ni} \triangleq \sum_{k=0}^{n-1} \binom{n-1}{k} \binom{N_u}{n} n a_i 2^{i-1} (k+N_u-n+1)^{i-1} \left(-1\right)^{k+1}$$

and $a_i = \frac{(2i)!(-1)^i}{2^{2i}(i!)^2}$, P_n is the allocated power to the $[n]^{th}$ ordered subcarrier and σ_v^2 is the variance of the additive white Gaussian noise.

Proof: Due to space limitations we provide only an outline of the proof of the above result. Let the random variable U_n be defined as the gain of the $[n]^{th}$ ordered subcarrier, ie $U_n \triangleq |H_{[n]}|^2$. From the independence property of the subcarrier gains described in the previous section, the probability density $f_{U_n}\left(u_n\right)$ corresponding to the gain of the $[n]^{th}$ ordered subcarrier can be obtained as,

$$p_{U_n}\left(u_n\right) = n \binom{N_u}{n} \left(1 - e^{-u_n}\right)^{n-1} \left(e^{-u_n}\right)^{N_u-n+1}.$$

A more elaborate discussion on order statistics can be found in [6]. The average probability of bit-error for BPSK transmission can be obtained in a straight forward manner as,

$$\phi_e(P_n) = \int_0^{\infty} Q\left(\sqrt{u_n\left(\frac{P_n}{\sigma_v^2}\right)}\right) p_{U_n}(u_n) du_n \tag{3}$$

Substituting the probability density $p_{U_n}\left(u_n\right)$ from (3), followed by further simplification results in the BER expression above. ■

The above polynomial form of expression in $\left(\frac{1}{P_n}\right)$ can be readily seen to have a diversity order of n [7], since the lowest power of $\left(\frac{\sigma_v^2}{P_n}\right)$ in the above polynomial is n.

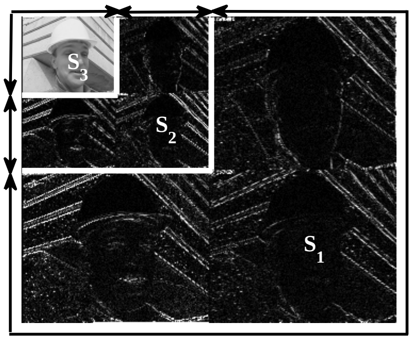

Fig. 1. Spatial decomposition of a frame into 3 layers

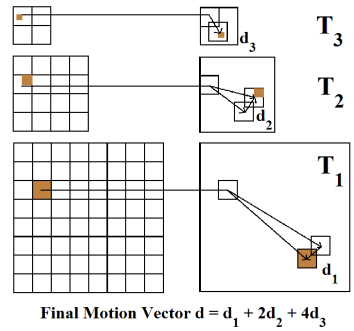

Fig. 2. Temporal decomposition of a frame into 3 layers

III. Diversity Based Hierarchical Allocation (DHA) for Video

In this section, we propose a scheme for hierarchical decomposition of the given video bit-stream into N_l spatio-temporal layers. For spatial layering, a discrete wavelet transform (DWT) [8] based multi-resolution decomposition can be employed to decompose the Intra-coded (I) frames into a spatial base layer S_{N_l} corresponding to the spatial low-frequency component and several enhancement layers $S_i, i \in 1, 2, ..., N_l - 1$ corresponding to spatial high-frequency components. An example of such a 3-layer DWT based decomposition is shown in Fig.1. The base-layer content S_3 of the I frame is shown in the top left corner of the transformed frame, which is naturally the layer of highest significance for frame decoding as compared to the enhancement blocks of the frame.

Similarly, in the temporal domain, HBMA [9] based hierarchical motion estimation and compensation can be employed to decompose the motion vectors of target P, B frames into corresponding base and enhancement components. The base layer motion component T_{N_l} is computed from a direct comparison of base resolutions of the target and anchor frames. Subsequently, these motion vector estimates at the lowest spatial resolution are propagated down the pyramidal structure of higher resolution images, with the progressive addition of enhancement motion vector estimates $T_j, j \in 1, 2, ..., N_l - 1$. The resulting motion vector is a sum of the corresponding motion vectors from each level of the pyramid. An example of such motion estimation is shown in Fig.2 and described in detail in [9]. The motion vector in the base layer T_{N_l}, clearly has a higher impact on video integrity when compared to the enhancement components $T_j, j \neq N_l$ at higher resolutions in the pyramid.

Thus, the video data can be naturally organized into hierarchical layers of varied significance. One can use this property of video advantageously for optimal ordered subcarrier diversity based power allocation. The user video sequence can be hierarchically decomposed such that $N_l = N_u$. The data layers with highest priority (S_{N_l}, T_{N_l}) are transmitted on the $[N_u]^{th}$ ordered subcarrier, i.e. the subcarrier with the largest gain. Each spatial and temporal layer of progressively lower significance S_l, T_l is transmitted on the $[l]^{th}$ ordered subcarrier (i.e. the lower ordered subcarriers). Thus, DHA combines spatio-temporal video layering with diversity based wireless communications to significantly minimize video distortion. The next section further minimizes the transmission distortion by optimally allocating the limited transmission power.

IV. Optimal and Closed Form Power Allocation

Employing the diversity based hierarchal video transmission scheme motivated in the above sections, we further minimize the overall video distortion through optimal power allocation amongst the N_u ordered subcarriers for a given user. For a given spatio-temporal video sequence $\mathcal{V}(x, y, t)$, the distortion $D_l(\mathcal{V})$ arising out of erroneous reception of the bit-stream corresponding to layer $l \in 1, 2, ..., N_u$ can be derived as,

$$D_l(\mathcal{V}) = D_l^S(\mathcal{V}) + D_l^T(\mathcal{V}), \tag{4}$$

where $D_l^S(\mathcal{V}), D_l^T(\mathcal{V})$, are the spatial and temporal distortion coefficients corresponding to layer l. The component $D_l^S(\mathcal{V})$ is defined as $E\left(\left|\left|\psi^{-1}(\Delta S_l)\right|\right|^2\right)$, where ψ is the wavelet employed for the spatial-layering DWT and ΔS_l is the error corresponding to the spatial layer S_l. The temporal distortion coefficient $D_l^T(\mathcal{V})$ is obtained as, $D_l^T(\mathcal{V}) = E\left(||\mathcal{V}(x, y) - \mathcal{V}(x + \Delta x_l, y + \Delta y_l)||^2\right)$, where $(\Delta x_l, \Delta y_l)$ is the MV component corresponding to layer l. Hence, employing the expression for the probability of error $\phi_e(P_l)$ given in (2), the mean overall distortion $D(\mathcal{V})$ for video transmission can be expressed as $D(\mathcal{V}) = \sum_{l=1}^{N_u} D_l(\mathcal{V}) \phi_e(P_l)$. The optimization problem for minimizing overall video distortion $D(\mathcal{V})$ for a video stream $\mathcal{V}$ is described as:

$$\begin{aligned} \min. \quad & \sum_{l=1}^{N_u} \sum_{i=l}^{\infty} \alpha_{li} D_l(\mathcal{V}) \left(\frac{\sigma_v^2}{P_l}\right)^i \\ \text{s.t.} \quad & \sum_{l=1}^{N_u} P_l = P_T \end{aligned}$$

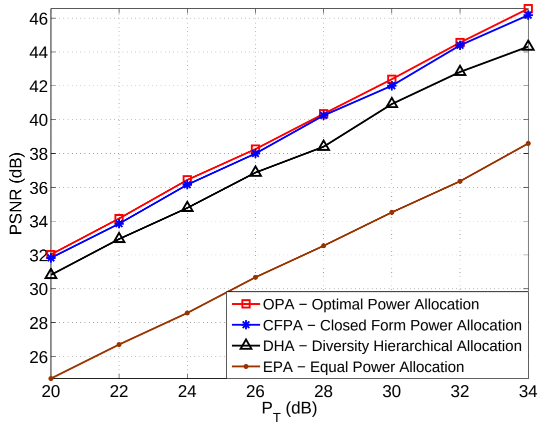

Fig. 3. PSNR plots for decoded Foreman video sequence

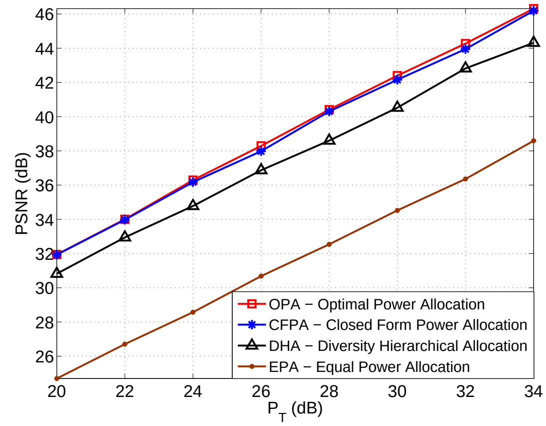

Fig. 4. PSNR plots for decoded Coastguard video sequence

$$P_l \geq 0,\ 1 \leq l \leq N_u \tag{5}$$

It can be observed that the distortion function $D(\mathcal{V})$ is a polynomial in $\left(\frac{1}{P_l}\right)$ and hence convex. The parameters $D_l(\mathcal{V})$, which are fixed for a video sequence $\mathcal{V}$ can be obtained through an offline computation. Further, the optimal power allocation (OPA) vector $\bar{P}^\star \triangleq \left[P_1^\star, P_2^\star, ..., P_{N_u}^\star\right]^T$ for this problem can be computed using the primal-dual interior point method [10]. By approximating the distortion cost employing the most significant term corresponding to the layer diversity order in the overall distortion function $D(\mathcal{V})$ one can obtain a simplified objective function, with much lower computational complexity. Thus, the optimization paradigm simplifies to,

$$\begin{aligned} \min. \ & \sum_{l=1}^{N_u} B_l(\mathcal{V}) \left(\frac{\sigma_v^2}{P_l}\right)^l \\ \text{s.t.} \ & \sum_{l=1}^{N_u} P_l = P_T \\ & P_l \geq 0,\ 1 \leq l \leq N_u \end{aligned} \tag{6}$$

where the constant $B_l(\mathcal{V}) \triangleq \alpha_{ll} D_l(\mathcal{V})$. The standard Lagrangian cost function $L(\mathcal{V}, \lambda, \mu)$ can be formulated for the above optimization problem as,

$$\sum_{l=1}^{N_u} B_l(\mathcal{V}) \left(\frac{\sigma_v^2}{P_l}\right)^l + \lambda \left(\sum_{j=1}^{N_u} P_j - P_T\right) - \sum_{k=1}^{N_u} \mu_k P_k \tag{7}$$

Assuming that $\mu_k = 0$, the KKT conditions for the above optimization problem can be readily obtained as,

$$\tilde{\lambda}^\star = \left(\frac{l B_l(\mathcal{V}) \left(\sigma_v^2\right)^l}{\left(\tilde{P}_l^\star\right)^{l+1}}\right) \geq 0,\ \tilde{\lambda}^\star \left(\sum_{j=1}^{N_u} \tilde{P}_j^\star - P_T\right) = 0$$

The optimal dual variable $\tilde{\lambda}^\star$ can be obtained as the solution of the polynomial equation,

$$\sum_{l=1}^{N_u} \left(\frac{l B_l(\mathcal{V}) \left(\sigma_v^2\right)^l}{\tilde{\lambda}^\star}\right)^{\frac{1}{l+1}} = P_T. \tag{8}$$

Hence, the optimal subcarrier power $\tilde{P}_l^\star$ corresponding to the modified optimization problem in (6) can be computed from the optimal dual variable $\tilde{\lambda}^*$ as,

$$\tilde{P}_l^\star = \left(\frac{l B_l(\mathcal{V}) \left(\sigma_v^2\right)^l}{\tilde{\lambda}^\star}\right)^{\frac{1}{l+1}}.$$

Thus, the closed form optimal power allocation (CFPA) vector $\tilde{P}^\star$ can be obtained as the solution of a polynomial root finding problem (8). The allocation computed from the above simplified optimization problem performs close to the OPA vector $\bar{P}^\star$ computed from (5) as demonstrated in the simulation results. Both the above schemes yield a significant performance enhancement compared to the sub-optimal equal power allocation (EPA) scheme corresponding to the power allocation vector $\hat{P} = \left(\frac{P_T}{N_u}\right)[1, 1, ..., 1]^T$ without diversity consideration.

V. Simulation Results

In our simulation setup, we employ the standard video test sequences from [11] for wireless video transmission. We consider an OFDMA system with $K = 4$ users. Each user is allocated $N_u = 2$ subcarriers, with BPSK modulated data transmitted on each subcarrier. We employ the level-1 Haar DWT for spatial decomposition of the intra-coded frame into $N_l = N_u = 2$ hierarchical layers. The lowest spatial frequency content, filtered in both the spatial dimensions forms the spatial base layer S_2 transmitted through the $[2]^{nd}$ ordered subcarrier, while the DWT filtered detail coefficients corresponding to S_1 are transmitted through the $[1]^{st}$ ordered subcarrier. Similarly, 2-level HBMA [9] algorithm is employed for hierarchical motion estimation as discussed in III. We employ a block size of 8×8 pixels and the search range is set to 4 pixels in each level of HBMA. After motion compensation of target frames, 2-layer Haar DWT is employed for spatial decomposition of corresponding residual frames. The distortion parameters $D_l(\mathcal{V}), 0 \leq l \leq N_l = N_u$ corresponding to the hierarchical layers of the video sequences are computed offline as described in section IV.

Fig. 5. Frame quality comparison of decoded Foreman video sequence from the transmission employing OPA (left) and EPA (right)

Fig. 6. Frame quality comparison of decoded Coastguard video sequence from the transmission employing OPA (left) and EPA (right).

Users 1-4 transmit the Foreman, Hall, Mobile and Coastguard video sequences respectively. We compare the performance of the optimal diversity schemes DHA, OPA and CFPA for video transmission with the sub-optimal equal power allocation (EPA). Fig.3 and Fig.4 demonstrate the PSNR of the decoded video streams corresponding to the received Foreman and Coastguard video sequences employing the different transmission schemes described above. It can be observed that the diversity based hierarchical allocation (DHA) (described in section III) where ordered subcarriers are allocated to the hierarchically decomposed layers of the video sequence (while allocating equal power to the ordered subcarriers) results in a 7dB performance enhancement over the sub-optimal EPA scheme. Thus, diversity order based hierarchical video allocation results in significant performance gains.

This PSNR performance can be further improved by 2dB (ie 9dB compared to EPA) through the optimal power allocation schemes OPA and CFPA described in section IV. Further, the performance of the OPA power allocation vector $\bar{P}^\star$ is very close to that of the optimal CFPA vector $\tilde{P}^\star$ computed as given in (9), thus demonstrating that the modified optimization problem in (6) for video distortion is an accurate approximation of the exact problem in (5). The superior performance of OPA over EPA is also visually illustrated by a comparison of the decoded received frames for the Foreman and Coastguard video sequences in Fig.5 and Fig.6 respectively. Thus, the DHA, OPA and CFPA schemes are ideally suited for video distortion minimization over OFDMA based wireless channels.

VI. Conclusion

In this work we proposed several order statistics based optimal power allocation schemes viz. DHA, OPA and CFPA for minimum distortion video transmission in OFDMA wireless systems. Since they are based only on order feedback, explicit feedback of CSI is not required, thus saving bandwidth and avoiding overheads on the reverse link. Simulation results show that the proposed optimal power allocation schemes are significantly superior to the equal power allocation (EPA) scheme for video transmission in an OFDMA system.

VII. Appendix

The result below describes a key property of the gains of the OFDMA subcarriers. The subcarriers occupy a bandwidth W, with inter-subcarrier spacing $\left(\frac{W}{N}\right)$.

Theorem 7.1: For a WSSUS Rayleigh fading channel, the correlation between subcarriers separated by $\left(\frac{W}{L}\right)$ is zero. Further, the correlation between subcarrier gains $H(m), H\left(m+r\right)$ separated by $r = \alpha\frac{N}{2L}$ subcarriers decreases as,

$$R_H(r) \triangleq \left|E\left(H(m)H^*(m+r)\right)\right| \leq \frac{1}{\alpha}R_H(0)$$

Proof: Proof is similar, as stated in [7]. ■

From the above result, it is clear that the correlation between the subcarrier gains decreases rapidly as $O\left(\frac{1}{\alpha}\right)$, becoming insignificant (< 0.5) for $r > \frac{N}{L}$. In other words, the gains of the subcarriers separated by more than the coherence bandwidth $\left(W_c = \frac{W}{L}\right)$ are essentially uncorrelated, and hence independent due to their Rayleigh fading nature. Thus, in an OFDMA system, each of the data streams transmitted through subcarriers with inter-subcarrier separation greater than $\left(\frac{W}{L}\right)$ can be assumed to experience independent Rayleigh fading.

References

[1] K.Kim, H.Kim, and Y.Han, "Subcarrier and power allocation in OFDMA systems," *Vehicular Technology Conference*, vol. 2, pp. 1058, Sep. 2004.

[2] J.Oh, J.Kim, and J.M. Cioffi, "Optimum power allocation and control for OFDM in multiple access channels," *Vehicular Technology Conference*, vol. 2, pp. 774 – 778, Sep. 2004.

[3] H. Ha, C. Yim, and Y.Y. Kim, "Cross-layer multiuser resource allocation for video communication over OFDM networks," *Comput. Commun.*, vol. 31, no. 15, pp. 3553–3563, 2008.

[4] S.Zhao, M.You, and L.Gui, "Scalable video delivery over MIMO OFDM wireless systems using joint power allocation and antenna selection," *IEEE International Workshop on Multimedia Signal Processing*, 2009.

[5] R.van Nee and R. Prasad, *OFDM for Wireless Multimedia Communications*, Artech House, Inc., 2000.

[6] H.A. David and H.N. Nagaraja, *Order Statistics*, John Wiley & Sons, Hoboken, New Jersey, U.S.A., 2003.

[7] David Tse and P. Viswanath, *Fundamentals of wireless communication*, Cambridge University Press, 2005.

[8] Alan. C. Bovik, *Handbook of Image and Video Processing*, Academic Press, Inc., Orlando, FL, USA, 2005.

[9] Y. Wang, Y. Zhang, and J. Ostermann, *Video Processing and Communications*, Prentice Hall PTR, 2001.

[10] Stephen Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[11] "Video sequences," http://media.xiph.org/video/derf/.