On the Equation $(\nabla u)^t H(u)(\nabla u) = G$

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Abstract

Let $n \in \mathbb{N}$, $n \geqslant 2$ and let $\Omega \subseteq \mathbb{R}^n$ be open. Let $H, G : \mathbb{R}^n \to \mathbb{R}^{n \times n}$ be of appropriate regularity. We discuss the existence of an immersion $u : \Omega \to \mathbb{R}^n$ of appropriate regularity, satisfying

$$(\nabla u)^t H(u)(\nabla u) = G \text{ in } \Omega. \tag{1}$$

We consider Cauchy, Dirichlet and Dirichlet-Neumann problems.

Equation (1) comes up in diverse contexts. When H (and hence G) is symmetric and positive definite, Equation (1) is connected to the problem of equivalence of Riemannian metrics. The symmetric case is also important in the non-linear elasticity theory because of its connection with the Cauchy-Green deformation tensor. When H (and hence G) is skew-symmetric, Equation (1) comes up in the context of the problem of equivalence of differential two-forms.

The aim of the talk is to present a survey of recent progress and advances made in the context of Equation (1). We shall also discuss the general case when H, G are neither symmetric nor skew-symmetric.