

# On the Convergence Order of the Finite Element Error in the Kinetic Energy for High Reynolds Number Incompressible Flows

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The kinetic energy of a flow is proportional to the square of the  $L^2(\Omega)$  norm of the velocity. Given a sufficient regular velocity field  $\mathbf{u}$  and a velocity finite element space with polynomials of degree  $r$ , then the best approximation error in  $L^2(\Omega)$  is of order  $r + 1$ . This talk will discuss which order of convergence for the velocity error in  $L^\infty(0, T; L^2(\Omega))$  can be proved with robust error estimates, i.e., with estimates where the constant does not depend on inverse powers of the viscosity. To fix ideas, the first part of the talk is devoted to evolutionary scalar convection-diffusion equations. In the second part, results for robust discretizations of the time-dependent incompressible Navier–Stokes equations will be surveyed. This survey covers as well inf-sup stable pairs of finite element spaces as pressure-stabilized discretizations.

This is joint work with Bosco García-Archilla (Sevilla) and Julia Novo (Madrid).