Indian Institute of Technology, Kanpur
Proposal for a new course

Course No.: MTH 6XX/7XX (PG level)

Course Title: Representation theory of quivers

Pre-requisites: Instructor's consent. However, knowledge of linear algebra (MTH204) and abstract algebra (MTH201) will be assumed.

Aims of the course: Around half a century ago, representations of quivers (directed graphs with the possibility of loops and parallel arrows) were originally introduced and studied by Gabriel, where he completely classified finite dimensional hereditary associative algebras that have only finitely many (iso-classes of) indecomposable modules using Dynkin diagrams that were previously used in the classification of finite dimensional semisimple Lie algebras. The utility of the quiver-theoretic techniques in the representation theory of finite groups was essentially demonstrated by Gel'fand and Ponomarev. In fact, representation theory of quivers became a synonym for the representation theory of finite-dimensional associative algebras during the development of Auslander-Reiten theory. The current century has seen exponential growth in the literature on quivers and related topics, especially due to connections with cluster algebras and topological data analysis.

The basic aim of this course is to give a self-contained introduction to this interesting branch of representation theory, where the necessary preliminaries from module theory over non-commutative rings and homological algebras will be covered. After successful completion of the course, the students will be equipped to read recent literature and tackle problems in the field.

Credits: 3-0-0-0 [9]

Semester: Odd Semester

Department/IDP: Mathematics and Statistics

Other departments which may be interested: PHY, CSE

Instructor: Amit Kuber

Course contents:

1. Quivers and representations: definitions, examples, category of representations, classification problem, special representations: simple, projective and injective [4 lectures]
2. Gabriel's theorem: root systems and Weyl groups, Dynkin diagrams, reflection functors, quadratic form, Coxeter functors [9 lectures]
3. Modules over associative algebras: associative algebras, Jacobson radical, local algebras, indecomposability, primitive idempotents, basic and connected algebras [4 lectures]
4. Bound quiver algebras: path algebras for quivers, arrow ideal, admissible ideals, bound quiver algebras, modules are representations and vice versa [4 lectures]
5. Auslander-Reiten quivers: ideals in module categories, radical of the module category and its powers, irreducible morphisms, A-R quiver examples [3 lectures]
6. Homological algebra: Hom-functors, duality, chain complexes and exact sequences, homology, projective and injective modules, resolutions and presentations, global dimension, Ext functor, tensor products and Hom-tensor adjunction [7 lectures]
7. Auslander-Reiten theory: Nakayama functor, A-R translate, stable categories, A-R sequences, A-R formulas, Coxeter transformation [7 lectures]
8. Representation types of finite-dimensional algebras [3 lectures]

Recommended texts/references:

1. Schiffler, Ralf. Quiver representations. (2014). CMS Books in Mathematics, Springer.

2. Auslander, M., Reiten, I., & Smalo, S. (1995). Representation Theory of Artin Algebras (Cambridge Studies in Advanced Mathematics). Cambridge: Cambridge University Press.
3. Assem, I., Skowronski, A., & Simson, D. (2006). Elements of the Representation Theory of Associative Algebras: Techniques of Representation Theory (London Mathematical Society Student Texts). Cambridge: Cambridge University Press.
4. Crawley-Boevey W., Lectures on representations of quivers. Available at https://www.math.uni-bielefeld.de/~wcrawley/quivlecs.pdf
5. Krause, H., Representations of quivers via reflection functors, Lecture notes available at https://arxiv.org/abs/0804.1428

Dated: 10/01/2025

Proposer: Amit Kuber

This course is approved/not approved

Convener, DPGC Maths and Stats

This course is approved/not approved

Chairman, SPGC